Model Order Reduction of Linear Time Interval System Using Stability Equation Method and a Soft Computing Technique

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Abstract. *This paper deals with a new method for model order reduction of linear continuous time interval system. This new method is based on the Kharitonov's theorem, the Stability equation method and the error minimization by Differential Evolution. The reduced order interval model is determined by using Kharitonov's polynomials, which make use of the Kharitonov's theorem and general form of the stability equation method for denominator, while the numerator is obtained by minimizing the integral square error between the transient responses of original and reduced order models using Differential Evolution algorithm. This method generates stable reduced order interval system if the original higher order system is stable and retains the steady-state value. The proposed method is illustrated with the help of typical numerical example considered from the literature.*

Keywords

Differential evolution, integral square error, interval system, Kharitonov's theorem, model order reduction.

1. Introduction

In general, the original system model is fairly complex and is of higher order. The understanding of the behavior of the system is difficult due to complexity. The analysis of a higher order is both tedious and costly. Therefore, the use of an order reduction makes it easier to implement analysis, simulations, and control system design. It has become necessary to use reduced order modeling techniques for the fundamental understanding of the systems characteristics. Model Order Reduction (MOR) is a branch of systems and control theory, for reducing their complexity, while preserving their input-output behavior. Order reduction methods are broadly classified into two types. Frequency domain order reduction methods are for the transfer function model. Time domain order reduction methods are for the state space model. Several methods are available in the literature for the order reduction of linear continuous systems in the time domain as well as the frequency domain. The reduced order model obtained in the frequency domain gives better matching of the impulse response with its higher order system.

Some of the most popularly used frequency domain order reduction methods are Pade approximation and continued fraction method. These are computationally fast and being able to match exactly the maximum number of system parameters to the reduced model. However, one of the disadvantages of these methods is that the stability of the reduced model is not guaranteed for stable higher order system. The effort has been devoted to developing stability preserving methods such as Routh stability criterion, Mihailov criterion, Hurwitz polynomial. The stability of these methods is achieved only by the loss of accuracy. Among these various model order reduction methods for stability preservation available in the literature, the stability equation method is one of the most popular techniques. The advantage of this method is that it preserves stability in the reduced model, if the original higher-order system is stable, and retains the first two time-moments of the system.

There are several methods available in the literature for order reduction, which are based on the minimization of the Integral Square Error (ISE) criterion. In [12], [13], the values of the denominator coefficients of the low order system are determined by some stability preservation methods and then the numerator coefficients of the low order systems are determined by minimization of the ISE using optimization technique.

Recently one of the most popular research fields has been "Evolutionary Techniques", inspired by the natural evolution of species. Evolutionary techniques have been successfully applied to solve numerous optimization problems. Differential Evolution (DE) was first proposed by Rainer Storn and Kenneth Price in 1996, it is a branch evolutionary algorithm. DE is a stochastic population based direct search algorithm. The advantages of DE are simplicity, accuracy, reasonable speed and the fact that it is a robust optimization method, which is, therefore, used to optimize real parameter value function. The differential evolution (DE) algorithm can be used to find approximate solution non-differentiable, nonlinear and multi-modal objective functions. The main difference between DE from other Evolutionary Algorithms (EA) is in the mutation and recombination phases. Another difference between DE and other EAs such as GA is that DE has the ability to search with floating point representation instead of binary representation that used in GA. DE employs a greedy selection. Also it has a minimum number of EA control parameters, which can be tuned effectively. The above methods are available for fixed systems only.

However, for many of systems the coefficients are fixed but uncertain within a finite range. Such systems are classified as interval systems. In [4] $\gamma - \delta$ Routh Approximation method for interval systems is proposed. The reduced model of interval system is unstable even when the original higher order interval system is stable. An improvement is proposed in [5] to the $\gamma - \delta$ Routh approximation for interval systems using the Kharitonov's polynomials such that the resulting interval Routh approximant is robustly stable. To improve the effectiveness of model order reduction many mixed methods have been proposed recently in [8], [9], and [10] based on interval arithmetic. Thus, the stability of the reduced order model is not guaranteed, if the original interval system is stable. In [17] and [18], the linear interval systems reduction techniques are presented using the Kharitonov's theorem to generate stable reduced order linear interval models. In [19], a reduction technique for linear interval systems using Kharitonov's polynomials and Routh Approximation is presented to generate a stable reduced order interval model. In [20], the reduced order interval model is obtained using Kharitonov's polynomials to retain stability and full impulse response energy of the higher order interval system in its reduced order interval model.

In this paper, model order reduction of interval systems is carried out by using the Kharitonov's theorem, stability equation method and differential evolution using ISE method. The denominator of the reduced model is obtained by the stability equation method and the numerator is obtained by minimizing integral square error between the transient response of original higher order system and the reduced order model pertaining to a unit step input. Thus, the stability is guaranteed for the reduced order system if the original higher order system is stable and the responses matching between original higher order system and the reduced order model.

2. Problem Formulation

Consider a higher order continuous time interval system given by the transfer function:

$$G_n(s) = \frac{N(s)}{D(s)} = \\ = \frac{[B_0^-, B_0^+] + [B_1^-, B_1^+]\cdot s + \cdots + [B_{n-1}^-, B_{n-1}^+]\cdot s^{n-1}}{[A_0^-, A_0^+] + [A_1^-, A_1^+]\cdot s + \cdots + [A_n^-, A_n^+]\cdot s^{n}}, \tag{1}$$

where $[A_i^-, A_i^+]$ for $i = 0, 1, \ldots, n$ are denominator coefficients of $G_n(s)$ with A_i^- and A_i^+ as lower and upper bounds of interval $[A_i^-, A_i^+]$ respectively, and $[B_i^-, B_i^+]$ for $i = 0, 1, \ldots, n-1$ are numerator coefficients of $G_n(s)$ with B_i^- and B_i^+ as lower and upper bounds of interval $[B_i^-, B_i^+]$ respectively.

It is proposed to obtain a reduced order interval model of the form:

$$G_r(s) = \\ = \frac{[b_0^-, b_0^+] + [b_1^-, b_1^+]\cdot s + \cdots + [b_{r-1}^-, b_{r-1}^+]\cdot s^{r-1}}{[a_0^-, a_0^+] + [a_1^-, a_1^+]\cdot s + \cdots + [a_r^-, a_r^+]\cdot s^{r}}, \tag{2}$$

where $[a_i^-, a_i^+]$ for $i = 0, 1, \ldots, r$ are denominator coefficients of $G_r(s)$ with a_i^- and a_i^+ as lower and upper bounds of interval $[a_i^-, a_i^+]$ respectively, and $[b_i^-, b_i^+]$ for $i = 0, 1, \ldots, r-1$ are numerator coefficients of $G_r(s)$ with b_i^- and b_i^+ as lower and upper bounds of interval $[b_i^-, b_i^+]$ respectively.

3. Proposed Method

Theorem 1 (Kharitonov theorem)**.** *An interval polynomial family* $K(s) = \sum_{i=0}^{n} [a_i^-, a_i^+] \cdot s^i$ *with invariant degree is robustly stable if its four Kharitonov polynomials are stable.*

According to the Thm. 1, every interval polynomial $K(s)$ is associated with the following four fixed parameter polynomials called Kharitonov polynomials. They are defined as:

$$\begin{aligned} K_1(s) &= a_0^- + a_1^- s + a_2^+ s^2 + \cdots + a_n^- s^n, \\ K_2(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + \cdots + a_n^- s^n, \\ K_3(s) &= a_0^+ + a_1^- s + a_2^- s^2 + \cdots + a_n^+ s^n, \\ K_4(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + \cdots + a_n^+ s^n. \end{aligned} \tag{3}$$

The interval system is stable if and only if its four Kharitonov polynomials satisfies Routh Hurwitz stability criterion.

3.1. Reduction Procedure

Consider a family of real interval transfer Eq. (1). The four fixed Kharitonov's transfer functions associated with $G_n(s)$ are given as:

$$\begin{aligned} G_n^1(s) &= \frac{N_n^1(s)}{D_n^1(s)} = \\ &= \frac{B_0^- + B_1^- \cdot s + B_2^+ \cdot s^2 + \cdots + B_{n-1}^- \cdot s^{n-1}}{A_0^- + A_1^- \cdot s + A_2^+ \cdot s^2 + \cdots + A_n^- \cdot s^n} = \\ &= \frac{B_{10}^- + B_{11}^- \cdot s + B_{12}^+ \cdot s^2 + \cdots + B_{1(n-1)}^- \cdot s^{n-1}}{A_{10}^- + A_{11}^- \cdot s + A_{12}^+ \cdot s^2 + \cdots + A_{1n}^- \cdot s^n}, \end{aligned} \tag{4}$$

$$\begin{aligned} G_n^2(s) &= \frac{N_n^2(s)}{D_n^2(s)} = \\ &= \frac{B_0^- + B_1^+ \cdot s + B_2^+ \cdot s^2 + \cdots + B_{n-1}^- \cdot s^{n-1}}{A_0^- + A_1^+ \cdot s + A_2^+ \cdot s^2 + \cdots + A_n^- \cdot s^n} = \\ &= \frac{B_{20}^- + B_{21}^+ \cdot s + B_{22}^+ \cdot s^2 + \cdots + B_{2(n-1)}^- \cdot s^{n-1}}{A_{20}^- + A_{21}^+ \cdot s + A_{22}^+ \cdot s^2 + \cdots + A_{2n}^- \cdot s^n}, \end{aligned} \tag{5}$$

$$\begin{aligned} G_n^3(s) &= \frac{N_n^3(s)}{D_n^3(s)} = \\ &= \frac{B_0^+ + B_1^- \cdot s + B_2^- \cdot s^2 + \cdots + B_{n-1}^+ \cdot s^{n-1}}{A_0^+ + A_1^- \cdot s + A_2^- \cdot s^2 + \cdots + A_n^+ \cdot s^n} = \\ &= \frac{B_{30}^+ + B_{31}^- \cdot s + B_{32}^- \cdot s^2 + \cdots + B_{3(n-1)}^+ \cdot s^{n-1}}{A_{30}^+ + A_{31}^- \cdot s + A_{32}^- \cdot s^2 + \cdots + A_{3n}^+ \cdot s^n}, \end{aligned} \tag{6}$$

$$\begin{aligned} G_n^4(s) &= \frac{N_n^4(s)}{D_n^4(s)} = \\ &= \frac{B_0^+ + B_1^+ \cdot s + B_2^- \cdot s^2 + \cdots + B_{n-1}^+ \cdot s^{n-1}}{A_0^+ + A_1^+ \cdot s + A_2^- \cdot s^2 + \cdots + A_n^+ \cdot s^n} = \\ &= \frac{B_{40}^+ + B_{41}^+ \cdot s + B_{42}^- \cdot s^2 + \cdots + B_{4(n-1)}^+ \cdot s^{n-1}}{A_{40}^+ + A_{41}^+ \cdot s + A_{42}^- \cdot s^2 + \cdots + A_{4n}^+ \cdot s^n}. \end{aligned} \tag{7}$$

The above Kharitonov's transfer function are, in general represented as:

$$G_n^1(s) = \frac{N_n^1(s)}{D_n^1(s)} = \frac{\sum_{j=0}^{n-1} B_{Ij} \cdot s^j}{\sum_{j=0}^{n} A_{Ij} \cdot s^j}, \tag{8}$$

where $I = 1, 2, 3, 4$.

1) Step 1

Determination of the denominator coefficients of lower order system for first Kharitonov transfer function by stability equation method. For $I = 1$:

$$\begin{aligned} G_n^1(s) &= \frac{N_n^1(s)}{D_n^1(s)} = \\ &= \frac{B_{10}^- + B_{11}^- \cdot s + B_{12}^+ \cdot s^2 + \cdots + B_{1(n-1)}^- \cdot s^{n-1}}{A_{10}^- + A_{11}^- \cdot s + A_{12}^+ \cdot s^2 + \cdots + A_{1n}^- \cdot s^n}. \end{aligned} \tag{9}$$

For first Kharitonov transfer function of the reduced order model is:

$$\begin{aligned} G_r^1(s) &= \frac{N_r^1(s)}{D_r^1(s)} = \\ &= \frac{b_{10}^- + b_{11}^- \cdot s + b_{12}^+ \cdot s^2 + \cdots + b_{1(r-1)}^- \cdot s^{n-1}}{a_{10}^- + a_{11}^- \cdot s + a_{12}^+ \cdot s^2 + \cdots + a_{1r}^- \cdot s^r}. \end{aligned} \tag{10}$$

For stable first Kharitonov transfer function $G_n^1(s)$, the denominator $D_n^1(s)$ of the Higher Order System (HOS) is bifurcated into even and odd parts in the form of stability equations as:

$$\left.\begin{aligned} D_e^n(s) &= A_{10} \prod_{i=1}^{m_1} \left(1 + \frac{s^2}{z_i^2}\right) \\ D_o^n(s) &= A_{11} s \prod_{i=1}^{m_2} \left(1 + \frac{s^2}{p_i^2}\right) \end{aligned}\right\}, \tag{11}$$

where m_1 and m_2 are the integer parts of $\frac{n}{2}$ and $\frac{n-1}{2}$ respectively and $z_1^2 < p_1^2 < z_2^2 < p_2^2 \cdots$ Now by discarding the factors with large magnitudes of z_i^2 and p_i^2 in Eq. (11), the stability equations for r^{th} order LOS are obtained as:

$$\left.\begin{aligned} D_e^r(s) &= A_{10} \prod_{i=1}^{m_3} \left(1 + \frac{s^2}{z_i^2}\right) \\ D_o^r(s) &= A_{11} s \prod_{i=1}^{m_4} \left(1 + \frac{s^2}{p_i^2}\right) \end{aligned}\right\}, \tag{12}$$

where m_3 and m_4 are the integer parts of $\frac{r}{2}$ and $\frac{r-1}{2}$, respectively. Combining these reduced stability equations and therefore proper normalizing it, the r^{th} order denominator of LOS is obtained as:

$$D_r^1(s) = D_e^r(s) + D_o^r(s) = \sum_{i=0}^{r} a_{1i} \cdot s^r. \tag{13}$$

Therefore, the denominator polynomial in Eq. (10) is now known, which is given by:

$$\begin{aligned} D_r^1(s) = a_{10} + a_{11} \cdot s + a_{12} \cdot s^2 + \cdots + \\ + a_{1(r-1)} \cdot s^{r-1} + a_{1r} \cdot s^r. \end{aligned} \tag{14}$$

2) Step 2

Determination of the numerator coefficients of the reduced model by Differential Evolution (DE). In this step, Differential Evolution (DE) is employed to minimize the objective function 'J', which is the error between the original higher order system and the reduced order system. Therefore it is represented in the form:

$$J = \int_0^\infty \left[y(t) - y_r(t)\right]^2 dt. \tag{15}$$

Mathematically, the integral square error can be represented as:

$$J = \sum_{t=0}^{M} \left[y(t) - y_r(t)\right]^2, \tag{16}$$

where, $y(t)$ is the unit step response of higher order and $y_r(t)$ is the unit step response lower order system at the t^{th} instant in the time interval $0 \leq t \leq M$, where M is to be chosen. The objective is to obtain a reduced order model, which is closely approximate original system. The objective function is to minimize ISE by using DE.

Differential evolution (DE) is a stochastic, population based direct search optimization algorithm introduced by Storn and Price in 1996 [15]. DE works with two populations; old generation and new generation of the same population. NP is the size of the population and it is adjusted. The population consists of real valued vectors with a dimension D that equals the number of design parameters/control variables. The population is randomly initialized within the initial parameter bounds. The three main operations carry optimization processes are: mutation, crossover and selection. In each generation, individuals of the current population become target vectors. For each target vector, the mutation operation produces a mutant vector. The crossover operation generates a new vector, called trial vector, by mixing the parameters of the mutant vector with those of the target vector. If the trial vector obtains a better fitness value than the target vector, then the trial vector replaces the target vector in the next generation.

3.2. Initialization

Define upper and lower bounds for each parameter:

$$X_j^L \leq X_{j,i,1} \leq X_j^U, \tag{17}$$

Randomly select the initial parameter values uniformly on the intervals $\left[X_j^L, X_j^U\right]$. For example, the initial value of the j^{th} parameter in the ith individual at the generation $G = 0$ is generated by:

$$X_{i,0}^j = X_{\min}^j + rand\,(0,1) \cdot \left(X_{\max}^j - X_{\min}^j\right) \\ j = 1, 2, \ldots D, \tag{18}$$

where NP is the population size, $rand\,(0,1)$ is a random number uniformly distributed between 0 and 1, D is the number of control variables.

3.3. Mutation

Mutation expands the search space. DE undergoes mutation operation after initialization. In mutation operation it produce mutant vector $V_{i,G}$, with respective to each individual $X_{i,G}$, so called target vector, in the current population via mutation strategy:

$$V_{i,G} = X_{i,G} + F\left(X_{best,G} - X_{i,G}\right) + \\ + F\left(X_{r1,G} - X_{r2,G}\right). \tag{19}$$

For a given parameter vector $X_{i,G}$ two vectors $X_{r1,G}$ and $X_{r2,G}$ are selected randomly such that the indices r_1, r_2 are distinct. The mutationfactor F is a constant-from $[0,2]$. $V_{i,G}$ is called the donor vector.

3.4. Crossover

Crossover incorporates successful solutions from the previous generation. After mutation, DE undergoes crossover. The trial vector $U_{i,G}$ is developed from the elements of the target vector, $X_{i,G}$, and the elements of the donor vector, $V_{i,G}$:

$$u_{i,G}^j = \\ = \begin{cases} v_{i,G}^j \text{ if } \left(rand_j\,[0,1) \leq \mathrm{CR}\right) \text{ or } \left(j = j_{rand}\right), \\ \qquad X_{i,g}^j \text{ otherwise.} \end{cases} \tag{20}$$

Elements of the donor vector enter the trial vector with probability CR (crossover rate) set to $[0,1]$.

3.5. Selection

The newly generated values of trail vectors exceed the corresponding upper and lower bounds; we initialize them randomly and uniformly within the pre-specified range:

$$X_{i,G+1}^j = \begin{cases} U_{i,g}^j \text{ if } f\left(U_{i,G}\right) \leq f\left(X_{i,G}\right), \\ \qquad X_{i,g}^j \text{ otherwise.} \end{cases} \tag{21}$$

The trail vector $X_{i,g}$ is compared with trail vector $U_{i,G}$ and the one with lowest function value is admitted to the next generation.

Therefore the four k^{th} order reduced Kharitonov's transfer function denominators are obtained by using stability equation method and the numerators are obtained by minimizing integral square error using Differential Evolution Algorithm. These four kth order reduced Kharitonov's transfer functions are represented as follows:

$$G_k^1(s) = \frac{b_{1k-1} \cdot s^{k-1} + b_{1k-2} \cdot s^{k-2} + \cdots + b_{10}}{a_{1k} \cdot s^k + a_{1k-1} \cdot s^{k-1} + \cdots + a_{10}}, \tag{22}$$

$$G_k^2(s) = \frac{b_{2k-1} \cdot s^{k-1} + b_{2k-2} \cdot s^{k-2} + \cdots + b_{20}}{a_{2k} \cdot s^k + a_{2k-1} \cdot s^{k-1} + \cdots + a_{20}}, \tag{23}$$

$$G_k^3(s) = \frac{b_{3k-1} \cdot s^{k-1} + b_{3k-2} \cdot s^{k-2} + \cdots + b_{30}}{a_{3k} \cdot s^k + a_{3k-1} \cdot s^{k-1} + \cdots + a_{30}}, \tag{24}$$

$$G_k^4(s) = \frac{b_{4k-1} \cdot s^{k-1} + b_{4k-2} \cdot s^{k-2} + \cdots + b_{40}}{a_{4k} \cdot s^k + a_{4k-1} \cdot s^{k-1} + \cdots + a_{40}}. \tag{25}$$

Finally the reduced order interval model is obtained by the following equation:

$$R_k(s) = \frac{\sum_{j=0}^{k-1} [\min(b_{Ij}), \max(b_{Ij})] \cdot s^j}{\sum_{j=0}^{k} [\min(a_{Ij}), \max(a_{Ij})] \cdot s^j}, \quad I = 1,2,3,4. \tag{26}$$

4. Numerical Example

Consider a higher order interval system from literature [4]:

$$G(s) = \frac{[54,74]s+[90,166]}{[1,1]s^4+[2.8,4.6]s^3+[50.4,80.8]s^2+[30.1,33.9]s+[0.1,0.1]}. \tag{27}$$

This higher order interval system can be represented as four fixed parameter Kharitonov transfer functions that are given as:

$$G_4^1(s) = \frac{54 \cdot s + 90}{s^4 + 4.6 \cdot s^3 + 80.8 \cdot s^2 + 30.1 \cdot s + 0.1}, \tag{28}$$

$$G_4^2(s) = \frac{74 \cdot s + 90}{s^4 + 2.8 \cdot s^3 + 80.8 \cdot s^2 + 33.9 \cdot s + 0.1}, \tag{29}$$

$$G_4^3(s) = \frac{54 \cdot s + 166}{s^4 + 4.6 \cdot s^3 + 50.4 \cdot s^2 + 30.1 \cdot s + 0.1}, \tag{30}$$

$$G_4^4(s) = \frac{74 \cdot s + 166}{s^4 + 2.8 \cdot s^3 + 50.4 \cdot s^2 + 33.9 \cdot s + 0.1}. \tag{31}$$

4.1. Step 1

Bifurcating the denominator of the above HOS in even and odd parts, we get the stability equations as:

$$D_e^4(s) = s^4 + 80.8 \cdot s^2 + 0.1, \tag{32}$$

$$D_o^4(s) = 4.6 \cdot s^3 + 30.1 \cdot s, \tag{33}$$

$$D_e^2(s) = \left(s^2 + 0.00123764272\right) \cdot \left(s^2 + 80.79876236\right), \tag{34}$$

$$D_o^2(s) = s \cdot \left(4.6 \cdot s^2 + 30.1\right). \tag{35}$$

Now by discarding the factors with large magnitude of z_i^2 and p_i^2 in $D_e^n(s)$ and $D_o^n(s)$ respectively, the stability equations for the second-order reduced model are given by:

$$D_e^2(s) = 80.79876 \cdot \left(s^2 + 0.00123764272\right), \tag{36}$$

Tab. 1: Typical parameter used by Differential Evolution for 1st Kharitonov's transfer function.

Name	Value
Population size	50
CR	0.8
F	0.5
Parameter 1: min,max	50,60
Parameter 2: min,max	80,90
Maximum generation	10

Tab. 2: Typical parameter used by Differential Evolution for 2nd Kharitonov's transfer function.

Name	Value
Population size	20
CR	0.8
F	0.5
Parameter 1: min,max	70,80
Parameter 2: min,max	80,90
Maximum generation	10

$$D_o^2(s) = 30.1 \cdot s, \tag{37}$$

$$D_2^1 = D_e^2(s) + D_o^2(s) = 80.9876 \cdot s^2 + 30.1 \cdot s + 0.1. \tag{38}$$

The reduced model is:

$$G_r^1(s) = \frac{b_{11} \cdot s + b_{10}}{80.79876 \cdot s^2 + 30.1 \cdot s + 0.1}. \tag{39}$$

Same as for remaining Kharitonov's transfer function the reduced order transfer functions are:

$$G_r^2(s) = \frac{b_{21} \cdot s + b_{20}}{80.79876 \cdot s^2 + 33.9 \cdot s + 0.1}, \tag{40}$$

$$G_r^3(s) = \frac{b_{31} \cdot s + b_{30}}{50.39802 \cdot s^2 + 30.1 \cdot s + 0.1}, \tag{41}$$

$$G_r^4(s) = \frac{b_{41} \cdot s + b_{40}}{50.39802 \cdot s^2 + 33.9 \cdot s + 0.1}. \tag{42}$$

4.2. Step 2

The numerator coefficients are obtained by minimizing integral square error using differential evolution.

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 1st Kharitonov's transfer function are (Tab. 1):

$$N_2^1(s) = 50.01287 \cdot s + 90. \tag{43}$$

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 2nd Kharitonov's transfer function are (Tab. 2):

$$N_2^2(s) = 74.01323 \cdot s + 90. \tag{44}$$

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 3rd Kharitonov's transfer function are (Tab. 3):

$$N_2^3(s) = 50.008167 \cdot s + 166. \tag{45}$$

Tab. 3: Typical parameter used by Differential Evolution for 3rd Kharitonov's transfer function.

Name	Value
Population size	50
CR	0.8
F	0.5
Parameter 1: min,max	50,60
Parameter 2: min,max	160,170
Maximum generation	10

Tab. 4: Typical parameter used by Differential Evolution for 4th Kharitonov's transfer function.

Name	Value
Population size	20
CR	0.8
F	0.4
Parameter 1: min,max	70,80
Parameter 2: min,max	160,170
Maximum generation	10

The reduced order numerator coefficients obtained by minimizing integral square error by DE for 4th Kharitonov's transfer function are (Tab. 4):

$$N_2^4(s) = 74.00109 \cdot s + 166. \tag{46}$$

The four reduced order Kharitonov's transfer functions are:

$$G_2^1(s) = \frac{54.01287 \cdot s + 90}{80.79876 \cdot s^2 + 30.1 \cdot s + 0.1}, \tag{47}$$

$$G_2^2(s) = \frac{74.01323 \cdot s + 90}{80.79876 \cdot s^2 + 33.9 \cdot s + 0.1}, \tag{48}$$

$$G_2^3(s) = \frac{54.00817 \cdot s + 166}{50.39802 \cdot s^2 + 30.1 \cdot s + 0.1}, \tag{49}$$

$$G_2^4(s) = \frac{74.00109 \cdot s + 166}{50.39801 \cdot s^2 + 33.9 \cdot s + 0.1}. \tag{50}$$

Therefore the reduced order interval system obtained by Eq. (26) is:

$$R_2(s) = \\ = \frac{[54.00817, 74.01323]s + [90, 166]}{[50.39801, 80.79876]s^2 + [30.1, 33.9]s + [0.1, 0.1]}. \tag{51}$$

Compare this with other method $\gamma - \delta$ method [4], Tab. 5:

$$R_2(s) = \\ = \frac{[0.9893, 3.7103]s + [0.5269, 1.8628]}{[1,1]s^2 + [0.3308, 0.7577]s + [0.00097579, 0.000251727]}. \tag{52}$$

Therefore the step responses of the original and reduced order Kharitonov's transfer functions are shown in Fig. 2, Fig. 4, Fig. 6 and Fig. 10 respectively.

Tab. 5: Comparison of integral square error for reduced order Khartonov's transfer function model.

Name	Integral Square Error Value	
	Proposed model	Model from B. Bandyopadhyay
1st Kharitonov's transfer function	0.216507	$3.41903 \cdot 10^9$
2nd Kharitonov's transfer function	0.082347	$4.8408 \cdot 10^9$
3rd Kharitonov's transfer function	1.20302	$3.4358 \cdot 10^{11}$
4th Kharitonov's transfer function	0.44852	$8.16029 \cdot 10^{10}$

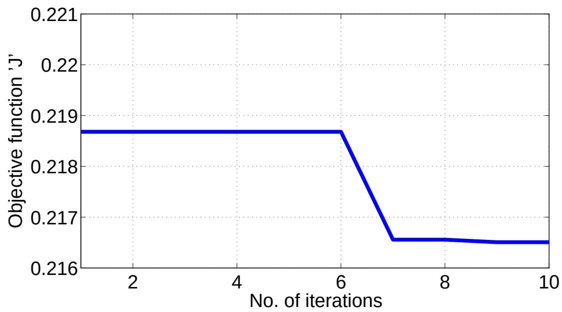

Fig. 1: Convergence graph (1st Kharitonov's TF).

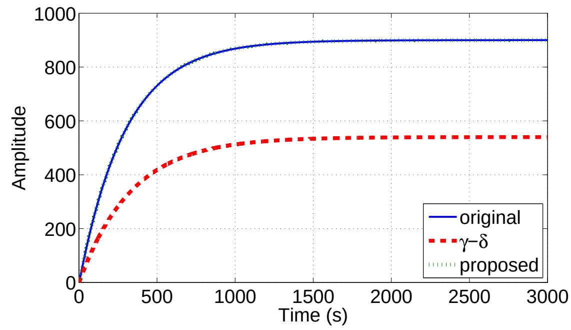

Fig. 2: Step Response (1st Kharitonov's TF).

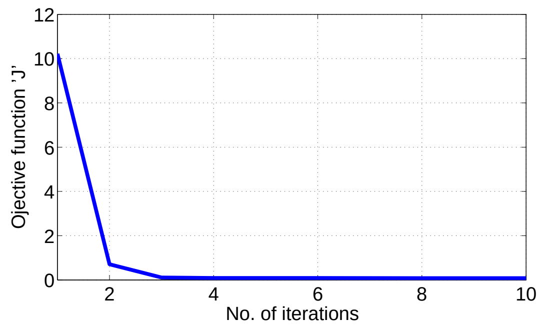

Fig. 3: Convergence graph (2nd Kharitonov's TF).

Tab. 6: Comparision of integral square error for reduced interval system model.

Method of order reduction	ISE for lower limit	ISE for upper limit
Proposed method	0.20018	0.50654
B. Bandyopadhyay [4]	$3.41738 \cdot 10^9$	$8.15954 \cdot 10^{10}$

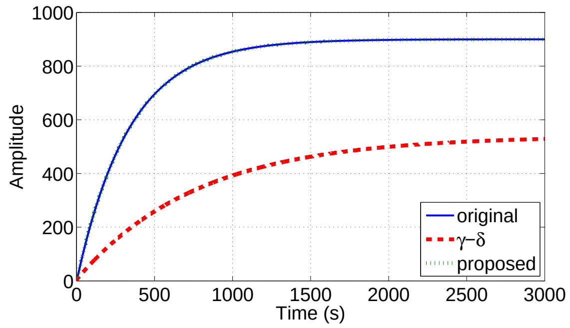

Fig. 4: Step Response (2$^{\text{nd}}$ Kharitonov's TF).

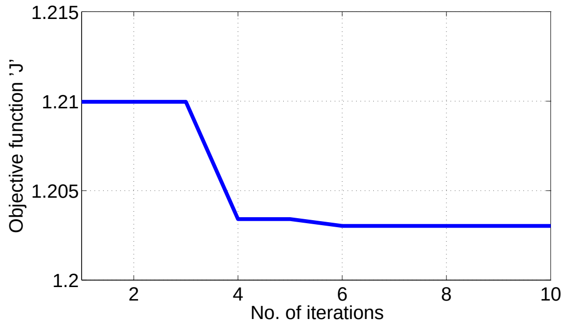

Fig. 5: Convergence graph (3$^{\text{rd}}$ Kharitonov's TF).

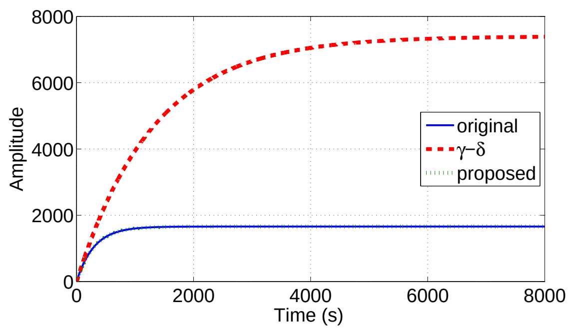

Fig. 6: Step Response (3$^{\text{rd}}$ Kharitonov's TF).

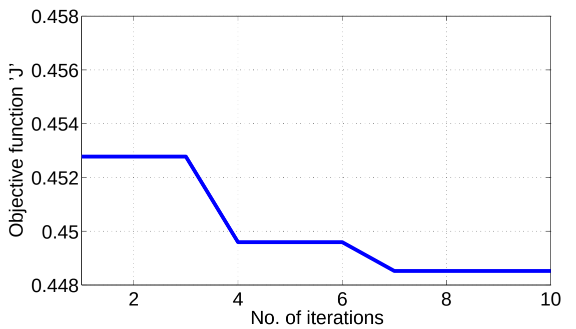

Fig. 7: Convergence graph (4$^{\text{th}}$ Kharitonov's TF).

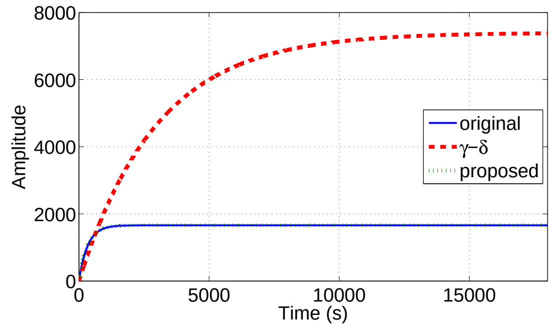

Fig. 8: Step Response (4$^{\text{th}}$ Kharitonov's TF).

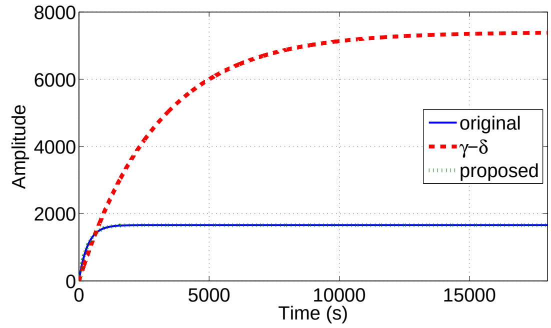

Fig. 9: Step Response (4$^{\text{th}}$ Kharitonov's TF).

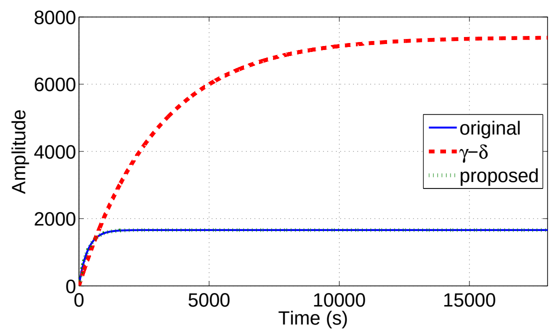

Fig. 10: Step Response (4$^{\text{th}}$ Kharitonov's TF).

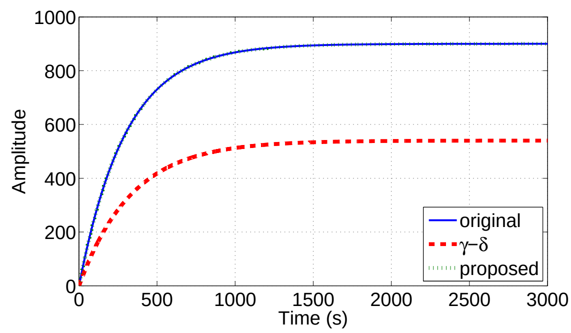

Fig. 11: Step Response for lower bounds.

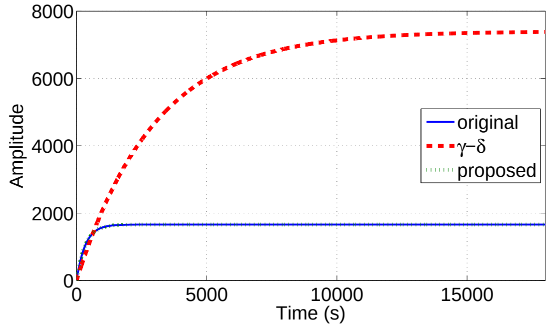

Fig. 12: Step Response for upper bounds.

5. Conclusion

In this paper, a new method for order reduction is proposed by combining the advantages of conventional method and an optimization technique. The reduced order interval system is obtained by using the Kharitonov's polynomial and the stability equation method for denominator coefficients, while numerator is obtained by minimising integral square error by using Differential Evolution. The use of interval arithmetic sometimes generates unstable reduced order model. Due to this, we use Kharitonov's polynomial to make the reduced order interval models robustly stable.

The reduced interval system preserves stability when the original higher order interval system is stable, and also has better matching response. Therefore the error is minimised between the original higher interval system and reduced order interval system.

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